RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [BATCH 2016-19]

B.A./B.Sc. FOURTH SEMESTER (January – June) 2018

Mid-Semester Examination, March 2018

Date : 16/03/2018

MATH FOR ECONOMICS (General)

Time : 1 pm – 2 pm

Paper : IV

Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

1. Answer **any two** questions : [2×2·5]

   a) If λ be an eigen value of a non-singular matrix A, then prove that λ^{-1} is an eigen value of A^{-1}.

   b) Verify Cayley-Hamilton theorem for the matrix A and use it find A^{-1}, where $A = \begin{pmatrix} 2 & 1 \\ -3 & 5 \end{pmatrix}$.

   c) Find the eigen values and the corresponding eigen vectors of the following real matrix $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$.

2. Answer **any one** question : [1×5]

   a) i) Define rank of a Linear transformation. [1]

   ii) Let P_i be the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$, with degree less equal to *i*. Let $I : P_2 \to P_3$ be a linear transformation defined by $I(f) = \int_0^x f(t)dt$. Find rank of I. [4]

   b) Let D be a linear operator on P_2 (See a(ii) for Definition) defined by $D(f(x)) = \frac{d}{dx}(f(x))$ and $\alpha = \{1, x, x^2\}$, $\beta = \{1+x, (1+x)^2, 1\}$ be two ordered basis of P_2. Find $[D]_\alpha$, $[D]_\beta$ $[D]_\alpha^\beta$. [5]

3. Answer **any one** question : [1×5]

   a) Find out the equilibrium of the following game using (i) iterated elimination of strictly dominated strategy (iii) the concept of Nash Equilibrium [2·5+2·5]

		Player B	
		Str. 1´	Str. 2´
Player A	Str. 1	10, 15	12, 23
	Str. 2	8, 10	7, 8

   b) Solve the following two person zero-sum game using maximin-minimax principle : [5]

		Player B		
		B_1	B_2	B_3
	A_1	6	3	–3
	A_2	–2	1	2
Player A	A_3	5	4	6

Group – B

4. Answer **any two** questions : [2×2]

a) Draw graphically the feasible region given by the L.P.P.

Maximize $z = x_1 + x_2$

Subject to $x_1 \le 2x_2$

$2x_1 \ge x_2$; $x_1, x_2 \ge 0$ [2]

b) Find the Basic feasible solutions (B.F.S) of the following two equations

$2x_1 + x_2 + 4x_3 = 11$

$3x_1 + x_2 + 5x_3 = 14$. [2]

c) Reduce the following L.P.P in its standard form, with non negative variables.

Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subject to $2x_1 - 3x_2 \le 3$

$4x_1 + 2x_2 - 4x_3 \ge 5$

$2x_1 + 0x_2 + 3x_3 \le 2$; $x_1, x_2, x_3 \ge 0$ [2]

d) Find the dual of the following L.P.P

Maximize $z = 3x_1 + 2x_2$

Subject to $3x_1 + 4x_2 \le 22$

$3x_1 + 2x_2 \le 16$

$x_2 \ge 3$; $x_1, x_2 \ge 0$ [2]

5. Answer **any one** question : [1×6]

a) Solve the following L.P.P graphically

Maximize $z = 150x + 100y$

Subject to $6x + 5y \le 60$

$4x + 5y \le 40$; $x, y \ge 0$ [6]

b) A firm manufactures three products A, B and C. The profits are Rs. 3, Rs. 2 and Rs. 4 for each unit of the products A, B and C respectively. The firm has two machines and below is the required processing time in minutes for each machine on each product. Machine X and Y have 2000 and 3000 machine minutes respectively.

		Product		
		A	B	C
Machine	X	4	3	5
	Y	2	3	4

The firm manufactures 100 A's, 200 B's and 50 C's but not more than 150 A's. Set up a L.P.P to maximize the profit. [6]

c) Given the L.P.P,

Maximize $z = 2x_1 + 3x_2 + 4x_3$

Subject to $x_1 - 5x_2 + 3x_3 = 7$

$2x_1 - 5x_2 \leq 3$

$3x_2 - x_3 \geq 5$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign. Formulate the dual of the L.P.P. [6]

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